Unions and Benefits: A Simple Analytical Framework

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(preliminary)

1 A model of trade unions and long term benefits.

This part presents a formal analysis of the non technical discussion in the main text.

We start with a monopoly trade union with "right to manage". There are two periods 1 and 2. In period 1, the trade union sets optimally, given its objective function, the level of wages, and long term benefits to be paid in period 2. The firm then decides its optimal level of employment in order to maximize its intertemporal profit. At the end of period 1, production takes place. In period 2, only a fraction of previously employed workers are still attached to the firm and receive therefore their promised long term benefits as chosen by the union in the first period.[1]

More precisely, consider a union with a total pool of M identical workers. Each worker cares about consumption over the two periods and has the following preferences: Consider the general case where each worker has the following preferences over the two periods

$$u_1(c_1) + \delta u_2(c_2)$$

with $u_i(.)$ is the intratemporal utility function of period i with the standard properties increasing concave with $u_i(0) = 0$. c_1 and c_2 are consumption levels in periods 1and 2 and δ is the discount rate.

[1]Hence benefits are of a defined benefits type rather than a defined contribution type.

Following Askildsen and Ireland (1997), we assume that there is some basic uncertainty concerning the future attachment of the worker to the firm in period 2. This may reflect the existence of a stochastic element affecting the worker's likelihood to stay in the firm in the next period. Alternatively, if we consider in this two period setting that workers work in period 1 and retire in period 2, the second period uncertainty may capture the uncertain longevity of the retired worker. Given such uncertainty, a worker will be concerned with the expected utility of consumption $Eu_2(c_2)$ in period 2. In period 1 each worker who is employed receives a wage rate w. This same worker gets the promise of a occupational pension s which is fullfilled in period 2. This specification therefore captures in a simple way the idea that workers may be more risk averse about uncertain benefits paid in the long term than about current wages paid today when $u_2(.)$ is more concave than $u_1(.)$.

A unemployed worker in period 1 receives an intertemporal utility payoff of V^R. Besides occupational benefits, we assume that there is a first tier public benefit system through which, independently from the employment status in period 1, an agent receives a transfer T in the second period of his life. This public system is financed in the economy by a payroll or income tax τ. Finally, in period 1, an unemployed worker receives an unemployment benefit b. Formally, we may write down the discounted expected utility of a union member when employed and when unemployed as: The discounted expected utility of a union member when employed and when unemployed is written as:

$$\begin{aligned} U(w(1-\tau), s, T) &= u_1(w(1-\tau)) + \delta[qu_2(s+T) + (1-q)u_2(T)] \\ &\text{and} \\ V^R(b, T) &= u_1(b) + \delta u_2(T) \end{aligned}$$

where q is the exogenous probability of attachment of the worker to the firm in the second period.

We take the standard view that, given a pool of workers M, the union wants to maximize the expected utility of the representative member which writes as:

$$V(L, w, s, \tau, T, b) = \frac{L}{M} U(w(1-\tau), s, T) + (1 - \frac{L}{M}) V^R(b, T)$$

where L is the number of employed workers in period 1

We consider that the firm sells a product which generates a revenue function in period 1 $R(L)$ increasing concave with $R(0) = 0$. Hence, given that only a fraction q ,of workers will remain attached to the firm in period 2, the firm's discounted expected profits can be written as

$$R(L) - wL - \frac{1}{1+r} sqL$$

with $\frac{1}{1+r}$ the discount factor. Finally let π^R be the reservation level of profits of the firm in order to be part of the relationship with the union. In the "right ot manage" model, the firm takes as given the current wage rate w and the long term benefit s and chooses its employment level in order ot maximize profits. This provides a demand function for labor $L^D(W)$ decreasing in $W = w + \frac{1}{1+r} sq$, the total expected discounted cost to hire a worker.

1.1 The determinants of the trade-off between current wages and deferred benefits

The problem of the monopoly trade union is then written as

$$\begin{aligned} & Max_{w,L,s} \qquad V(L, w, s, \tau, T, b) \\ L &= L^D(W) \\ W &= w + \frac{1}{1+r} sq \end{aligned}$$

This problem is easily decomposed into two steps. First, we may solve the optimal mix between current wages w and deferred payments s for a typical employed worker, given a fixed expected discounted cost of hiring W for the firm. Second, we may determine the trade union's optimal expected discounted labor cost

Optimal trade-off between current wages and deferred benefits
More precisely the first step can be written as the following problem:

$$Max_{w,s} \quad u_1(w(1-\tau)) + \ \delta[qu_2(s+T) + (1-q)u_2(T)]$$

$$w + \frac{1}{1+r} sq = W$$

which, after a change of variable $w' = w(1-\tau)$ and $s' = s + T$, can be rewritten as:

$$Max_{w,s} \quad u_1(w') + \ \delta[qu_2(s') + (1-q)u_2(T)]$$

$$\frac{1}{1-\tau}w' + \frac{1}{1+r}s'q = W + \frac{1}{1+r}qT$$

The optimal wage and long term benefit are given by the standard marginal condition

$$\frac{u_1'(w')}{u_2'(s')} = \frac{\delta(1+r)}{1-\tau} \tag{1}$$

and the budget constraint. From this we find the net wage $w' = w(1-\tau) = w^*(q, W, \tau, T)$ and total benefits $s' = s + T = s^*(q, W, \tau, T)$ and the indirect utility function of an employed worker $U(w^*, s^*) = U^*(q, W, \tau, T)$.

The solution of this problem is illustrated in figure 1. In the plane (w', s') of after tax current wages $w' = w(1-\tau)$ and total long term benefits $s' = s + T$, iso-utility curves $u_1(w') + \ \delta qu_2(s') = cst$ are drawn with the usual decreasing convex shape. The "budget line" BB under which the employed worker expected utility is maximized is given by the expression $\frac{1}{1-\tau}w' + \frac{1}{1+r}s'q = W + \frac{1}{1+r}qT$. It goes through point H $(W(1-\tau), T)$ with a slope $\frac{1-\tau}{1+r}q$. The optimal allocation (w'^*, s'^*) of after tax current wages and total long term benefits is given at point A where the budget line" BB is tangent to the optimal iso-utility curve $U^* = U(w^*, s^*) = U^*(q, W, \tau, T)$.

An increase in q induces a clockwise rotation around point B of the "budget line" (see figure 2). At the same time, it makes the iso-utility curves of the worker steeper. In general, the rotation of the "budget line" has the three traditional substitution, income and wealth effects on (w'^*, s'^*). The substitution effect tends to increase w'^* and to reduce s'^*. Under standard "normality" assumptions on current and future consumption, the income effect tends to reduce both w'^* and s'^*. The wealth effect (coming from the term $\frac{1}{1+r}qT$ in the "budget" line) on the contrary induces an increase in w'^* and s'^*. The fact that the utility curves become steeper as a result of an increase in q, on the other hand, induces a decrease in w'^* and an increase in s'^* as future consumption is more valued by the worker.

It is easy to see that the substitution effect and steeper utility slope effects cancel out for total benefits and that the rotation around point H of the "budget line" leads along the locus determined by equation (1) to a decrease in the net wage rate w' and total benefit s' (see figure 2). Obviously the effect is similar for gross wages w and occupational benefits s.

Similarly an increase in the discounted expected labor cost W or an increase in public benefits T shifts out the "budget line" and implies an increase both in net and gross wages and total benefits. However it is easy to see that

$$\frac{\partial s'}{\partial T} = \frac{u"_1(w')}{u"_1(w') + \frac{\delta}{q}\frac{(1+r)^2}{(1-\tau)^2}u"_2(s')} \leq 1$$

Hence occupational benefits $s = s' - T$ decrease with public transfers T. There is some crowding out of public benefits on occupational benefits.

Finally an increase in the payroll tax τ has the usual substitution and income effects with a resulting decrease in the net wage rate w' and a total ambiguous effect on benefits s' and s.

- *The quasi linear case:*

  An interesting tractable case is the quasi linear case: $u_1(c_1) = c_1$ and $u_2(c_2) = u(c_2)$

It is easy to get:

$$w^* = w^*(q, W, \tau, T) = W - \frac{1}{1+r}q[u'^{-1}(\frac{1-\tau}{\delta(1+r)}) - T]$$

and

$$s^* = u'^{-1}(\frac{1-\tau}{\delta(1+r)}) - T$$

and the indirect utility function of an employed worker $U(w^*, s^*) = U^*(q, W, \tau, T)$ with:

$$U^*(q, W, \tau, T) = W(1-\tau) + \frac{1}{1+r}qT(1-\tau) - \frac{1-\tau}{1+r}q[u'^{-1}(\frac{1-\tau}{\delta(1+r)})] + \delta q u[u'^{-1}(\frac{1-\tau}{\delta(1+r)})]$$

In the simple quasi linear specification,as all income and wealth effects fall on w'^*, there is no net effect of q or W on total benefits s'^*.

Employment and labor costs:

The second step of the union problem can be written as:

$$Max_W \quad V(W, q, \tau, T, b) = \frac{L^D(W)}{M}U^*(q, W, \tau, T) + (1 - \frac{L^D(W)}{M})V^R(b, T)$$

$$R(L^D(W)) - WL^D(W) \geq \pi^R$$

which collapses to the fairly standard problem of the monopoly trade union. Given that there is a participation constraint of the firm, the standard labor cost solution is given by $W = W^* = Min\left(\widetilde{W}(q,\tau,T,b), \overline{W(\pi^R)}\right)$ where $\widetilde{W}(q,\tau,T,b)$ is the interior solution of first order condition

$$\frac{dL^D(W)}{dW}[U^*(q,W,\tau,T) - V^R(b,T)] + L^D(W)\frac{dU^*(q,W,\tau,T)}{dW} = 0 \qquad (2)$$

and $\overline{W(\pi^R)}$ is the labor cost level which makes the firm's participation constraint binding:

$$R(L^D(\overline{W(\pi^R)})) - \overline{W(\pi^R)}L^D(\overline{W(\pi^R)}) = \pi^R$$

$>$From this it is straightforward to recover wages, benefits and employment:

$$\begin{aligned} w &= w^*(q,W^*,\tau,T)/(1-\tau) \\ s &= s^*(q,W^*,\tau,T) - T \\ L &= L^D(W^*) \end{aligned}$$

The solution is depicted in figure 3 in the space (W,L) of total expected labor cost W and employment level L.The iso-utility curves of the trade union are depicted by the curve UU while the labor demand $L = L^D(W)$ is the locus of vertical tangents of the iso-profit curves $\pi\pi$ of the firm. Given that there is a participation constraint of the firm, the standard labor cost solution is given by $W = W^* = Min\left(\widetilde{W}(q,\tau,T,b), \overline{W(\pi^R)}\right)$ where $\widetilde{W}(q,\tau,T,b)$ is associated to the "monopoly trade union equilibrium" point E at which the optimal iso-utility curve of the union is tangent to the labor demand curve and $\overline{W(\pi^R)}$ is the labor cost level at which the firm's participation constraint is bind.

Comparative statics

It is interesting to see how, in this simple framework, wages, occupational pensions and employment change with the probability of attachment to the firm (or concern for long term benefits) q, unemployment benefits b, reservation profit level π^R, public pensions T and payroll tax rates τ.It is first clear that, with our quasi linear specification, pensions are unaffected by changes in q, b or π^R.

An increase in unemployment benefits b implies, in a standard fashion, an increase in the equilibrium intertemporal labor cost W^* (at least as long as the participation constraint of the firm is not binding). This, in turn, is is associated with an outward shift of equilibrium wages w and a reduction in employment L. Similarly, a positive change in the firm's reservation profit level π^R implies a decrease in W^* (when the firm's participation constraint is binding) with a reduction in w and an increase in employment L.

The impact of a change in q is less straightforward (see figure 4). Getting back to the definition of the indirect intertemporal utility $U^*(q, W, \tau, T)$ of an employed worker and the intertemporal payoff $V^R(b, q, T)$ of an unemployed one, it can be seen that an increase in q will induce flatter iso-utility curves for the trade union. A higher probability of future attachment to the firm increases the worker's value to be employed today by the firm, reducing therefore the union's incentive to choose a high total labor cost W. Under reasonable concavity assumptions, this implies that the new tangency point at E' between the labor demand curve and the union 's iso-utility is on the north west direction of the initial equilibrium point E. This is associated with a lower expected cost of labor $\widetilde{W}(q, \tau, T, b)$ and a higher employment level. Given that the direct impact of q on w is negative, this implies that the total impact of an increase in q is to reduce current wages

Formally, differentiation of the indirect intertemporal utility of an employed and an unemployed worker provides:

$$\begin{aligned}\frac{\partial U^*(q, W, \tau, T)}{\partial q} - \frac{\partial V^R(b, q, T)}{\partial q} &= \frac{\partial U^*(q, W, \tau, T)}{\partial q} \\ &= \delta \left[u_2(s') - u_2(T) + (T - s') u_2'(s') \right] > 0\end{aligned}$$

and

$$\frac{\partial^2 U^*(q, W, \tau, T)}{\partial q \partial W} = \delta (T - s') u_2"(s') \frac{ds'}{dW} > 0$$

as workers are risk averse (ie. $u_2"(s) < 0$). Therefore, when the union problem is well behaved (ie. $\frac{\partial^2 V}{\partial W^2} < 0$), differentiation of the first order condition (2) provides

$$\frac{\partial \widetilde{W}(q, \tau, T, b)}{\partial q} = -\frac{\frac{\partial^2 V}{\partial W \partial q}}{\frac{\partial^2 V}{\partial W^2}} < 0$$

with

$$\frac{\partial^2 V}{\partial W \partial q} = \frac{dL^D(W)}{dW}[\frac{\partial U^*(q,W,\tau,T)}{\partial q} - \frac{\partial V^R(b,q,T)}{\partial q}] + L^D(W)\frac{\partial^2 U^*(q,W,\tau,T)}{\partial q \partial W} < 0$$

Hence when the condition

$$\frac{dL^D(W)}{dW}\left[u_2(s') - u_2(T) + (T-s')u_2'(s')\right] + L^D(W)(T-s')u_2"(s')\frac{ds'}{dW} < 0$$

is satisfied, $\widetilde{W}(q,\tau,T,b)$ is decreasing in q and the trade union solution W^* is decreasing in q. This is obviously satisfied for the quasi linear case.

The equilibrium employment level is then increasing in q. Given that the direct impact of q on w is negative, this implies that the total impact of an increase in q is to reduce current wages

Similarly, it can be seen that an increase in public pensions T will induce steeper trade union iso-utility curves (see figure 5). The intuition is the fact that, as public transfers are not conditional on the employment status, they tend to reduce the relative value for a worker to be employed in period 1. This in turn increases the union's incentive to choose a high total labor cost W with an associated lower employment level. Hence $\widetilde{W}(q,\tau,T,b)$ is increasing in T implying that the equilibrium intertemporal labor cost W^* is also weakly increasing in T.

Formally, the effect of an increase in public pensions T on labor costs W will have the sign of $\frac{\partial^2 V}{\partial W \partial T}$:

$$\frac{\partial^2 V}{\partial W \partial T} = \frac{dL^D(W)}{dW}[\frac{\partial U^*(q,W,\tau,T)}{\partial T} - \frac{\partial V^R(b,q,T)}{\partial T}] + L^D(W)\frac{\partial^2 U^*(q,W,\tau,T)}{\partial T \partial W}$$

But

$$\frac{\partial U^*(q,W,\tau,T)}{\partial T} - \frac{\partial V^R(b,q,T)}{\partial T} = q\delta\left[u'(s') - u'(T)\right] < 0$$

and

$$\frac{\partial^2 U^*(q,W,\tau,T)}{\partial T \partial W} = q\delta u"(s')\frac{ds'}{dW} < 0$$

Hence $\widetilde{W}(q,\tau,T,b)$ is increasing in T when:

$$\frac{dL^D(W)}{dW}\left[u'(s') - u'(T)\right] + L^D(W)u"(s')\frac{ds'}{dW} > 0$$

which is again satisfied for the quasi linear case. This implies that the equilibrium intertemporal labor cost W^* is also weakly increasing in T. Gross and net current wages will consequently also increase with T.

In the general case with preferences which are not quasi linear, total pensions s' are also likely to increase. Occupational pensions are partially crowded out by public pensions through the direct effect of T. At the same time there is an additional income effect coming from the fact that the equilibrium intertemporal labor cost W^* does increase. The whole effect of public pensions on occupational pensions is therefore on a priori grounds ambiguous. When the impact of public pensions on wages and labor costs is strong enough (resp. weak enough), the second effect is likely to dominate (resp. be dominated) and one there is a complementarity (substituability) between public pensions and occupational pensions.

Finally, let us close this section by investigating the effect of an increase in the payroll tax rate τ. As is straightforward to see, the indirect utility of an employed worker $U^*(q, W, \tau, T)$ is obviously negatively related to the payroll tax rate. This implies the standard result (Alesina and Perotti (1998) that the union passes to the employer some fraction of the fiscal burden and that the union determined discounted labor cost $\widetilde{W}(q, \tau, T, b)$ is increasing in τ. Thus an increase in the payroll tax, everything else being equal, has a negative impact on employment L. The effect on current wages is generally ambiguous.The direct impact of an increase in τ is negative but there is a additional positive income effect coming from the increase in W. For occupational pensions, the positive substitution effect and the induced income effect through the increase in W, imply that, in general, they are increasing in the payroll tax τ.

2 Bargaining

The simple model of the previous section can be extended along several dimensions. First one may generalize the analysis to situations where the union bargains with the firm on wages and benefits, while the firm keeps its "right to manage" on employment decisions. Figure 6 reflects such a situation.in the plane (W, L) of discounted expected labor cost and employment. The downward sloping demand curve of the firm.is represented by $L^D(W)$ while typical isoprofit curves $\pi\pi$ and trade union iso-utility curves VV are also drawn on the graph. A bargaining game on wages and benefits with "right to manage" will essentially pick a point on the labor demand curve between the reservation level curve of the firm $\pi^R\pi^R$ and the reservation discounted labor cost of the union W^R corresponding its reservation value V^R.Once such a solution (W^B, L^B) is found, it is easy to recover wages and benefits as $w^B = w^*(q, W^B)$ and $s^B = s^*(q, W^B)$.It should be clear by then that all the previous comparative statics results on b, π^R, q, τ, T are qualitatively the same as in the monopoly model of trade union.

In a similar fashion, one can consider the situation of full bargaining between the firm and the union on the three dimensions current wages, occupational pensions and employment (w, s, L). This is again represented in figure 6. The main difference with the previous case is simply that now the optimal point has to be picked up on the "contract" curve CC of efficient allocations (W, L) between the union and the firm with an outcome implying more employment L_N, lower expected discounted labor costs W_N, lower wages and benefits than in the "right to manage" case . Again, one will get the same qualitative comparative statics as in the case of the monopoly union model.

3 Political economy considerations within the union

So far we considered a framework in which all workers were identical. Obviously, one of the important features of non wage and benefit policies for unions is the fact that they do not affect all workers the same way. In this section, we introduce some heterogeneity across workers' preferences with respect to long term benefits and employment. Various dimensions may be captured by such differentiation: age, seniority or degree of attachment to the firm. As long as unions can be viewed as groups taking collective decisions along more or less democratic rules, this introduces a number of interesting political economy issues within the union. What are the points of conflicts or convergence between young and old workers inside the union?. Which kind of preferences of the various workers are represented in the objective function of the union? What is the effect of workers heterogeneity on wages, pensions, employment, membership? To these questions we turn now.

3.1 Introducing young and old workers

We start first by amending our preceding two period framework and allow for the fact that there are now young and old workers in the union. Young workers may work in the first period (period 1) of their life and retire in the second period (period 2) (if still alive). Old workers, in fixed number L_0, are retirees and live only in period 1. They do not work and receive benefits s_0 in that period. At the beginning of period 1, the "monopoly" union chooses the current wage rate of young workers w_y, their future occupational benefits s_y to be paid in period 2. Given this, the firm chooses the employment level $L_{y.}$ In period 2, benefits s_y are paid to the workers who were previously employed.

We will first assume that the structure of political representation within the union is such that the union's objective function will be the expected discounted utility of young workers in period 1, given some veto power of the old workers to receive what they were promised to receive, namely their benefits s_0.

Given this, and noting $W_y = w_y + \frac{1}{1+r} s_y q$ the discounted labor cost of a young worker in period 1, the problem of the union can be written as:

$$Max_W \quad \frac{L^D(W_y)}{M} U^*(q, W_y, \tau, T) + (1 - \frac{L^D(W_y)}{M}) V^R(q, b, T)$$

$$R(L^D(W_y)) - W_y L^D(W_y) - s_0 L_0 \geq \pi^R$$

With our assumptions, the only difference with the previous section is the fact that now the firm has to generate a large enough surplus top be sure that benefits to old workers can be paid in period 1. Hence the minimum surplus that has to be left over for the firm and old workers is $\pi^R + s_0 L_0$. Obviously, the solution of this problem is simply (with the same notations as before) $W_y = W_y^* = Min\left(\widetilde{W}(q, \tau, T, b), \overline{W(}\pi^R + s_0 L_0)\right)$.

This is represented in figure 7. The first quadrant represents the, by now, usual monopoly union equilibrium in terms of the young worker employment level L_y and the discounted labor cost W_y. The iso-utility curve of the young worker EV_y is tangent to the demand function at the point $\widetilde{W}(q, \tau, T, b)$. The second quadrant plots the labor cost $\overline{W(}\pi^R + s_0 L_0)$ such that the participation constraint of the firm is binding, as a function of the size L_0 of old workers in the union. This is a downward sloping relationship as a larger current surplus and a lower associated labor cost W is necessary to accommodate higher expenditures of the firm on occupational pensions of old workers. The equilibrium discounted labor cost of young workers W_y^* is then also depicted in bold. It is first fixed at the level $\widetilde{W}(q, \tau, T, b)$ and then decreasing with L_0 after some threshold $\overline{L}_0$. From this it is easy to get that after this threshold level, employment of young workers L_y is positively related to the size of old workers in the union. The intuition.is simple. A larger volume of occupational pensions paid to old workers induces the union to moderate its demand on labor costs W for the next generation of workers in order to generate a high enough firm's surplus. This, in turn, is associated to a larger fraction of employed workers in period 1.

Still, it should be noted that such a situation is not in the interest of the young workers. As can be seen in figure 7, when L_0 becomes larger than $\overline{L}_0$, the equilibrium expected discounted utility $EV_y^T$of a young worker is decreasing with the size of old workers in the union. Obviously, this aspect has interesting implications for the dynamics of membership inside the union. Consider for instance now that trade union membership M is endogenous and that young workers do enter in the union when their expected discounted utility in the union is larger than some reservation level $\overline{V}$. The preceding discussion implies that young workers will be less likely to enter the union, the larger the size of retired old workers.

3.2 Voting on benefits and wages

In the preceding framework, the decision making process within the union on current wages and occupational pensions was quite simple. The objective function of the union represented the preferences of the young workers under the constraint of veto power of the old retiree union members who did not work. Following Askildsen and Ireland (1997), we assume now that workers inside the union now are continuously differentiated according to their relative atttachment to the firm or more broadly their relative evaluation between current wages and long term benefits and that they vote to decide the union position on wages, benefits and employment.

Indeed, suppose that the parameter q is distributed in the union along a distribution $f(.)$ with mean $\overline{q}$ and median q^m. Abstracting for simplicity from taxes and denoting public benefits to former unemployed as T_b. Also we restrict ourselves to the case of quasi linear preferences:

$$U(w, s, q) = w + \delta q u(s) \text{ and } V^R(b, q) = b + \delta q u(T_b)$$

The expected utility of a worker with attachment q can then be written as:$\frac{L}{M}U(w,s,q)+(1-\frac{L}{M})V^R(b,q)$. The firm's average profit in such a context can be written as

$$E\pi = R(L) - wL - \frac{1}{1+r}s\overline{q}L$$

implying an average discounted labor cost for each hired worker of $W = w + \frac{1}{1+r}s\overline{q}$.

As we have now some degree of heterogeneity across workers, we need to model the political mechanism by which collective decisions will be taken within the union. Let us consider first the case where wages and benefits are decided by simple majority voting. A typical technical problem in term of the determination of the political equilibrium is the fact that voting has to be on two dimensions (w, s). In order it alleviate this issue, assume further that there is sequential voting. First unions members vote on the discounted labor cost W faced by firms (or equivalently on the employment level $L = L^D(W)$ in the"right to manage" specification) and then they vote on how to allocate this cost W between current wages and future benefits. We solve the game, as usual, by backward induction

Let us look therefore at the second stage of this collective decision mechanism. For a given $L = L^D(W)$ and $W = w + \frac{1}{1+r}s\overline{q}$, the typical problem of a worker of type $q$can now be written as:

$$Max_{w,s} \quad w + \ \delta q u(s)$$

$$w + \frac{1}{1+r}s\overline{q} = W$$

The solution of this problem is illustrated in figure 8 at the tangency point of the iso-utility curve UUq of a typical worker and the "budget line" $BB\overline{q}$ providing the optimal current wage $w = w^*(\frac{q}{\overline{q}}, \overline{q}, W)$ and benefits $s = s^*(\frac{q}{\overline{q}})$ and the indirect utility function of an employed worker $U(w^*, s^*, q) = U^*(q, \overline{q}, W)$ (see the appendix). An increase in q induces steeper indifferences curves for the worker without changing the "budget line". This leads in turn leads to a lower current wage w^* and a higher longer term benefit s^* at the optimal tangency point. Obviously the indirect utility of an employed worker of type q is increasing in q. An increase in the average value $\overline{q}$ induces a clockwise rotation of the "budget line" around point A. Because of conflicting substitution and income effects, the impact of such a change on current wages w is ambiguous. In the quasi linear specification, an increase in $\overline{q}$

has only a negative substitution effect on benefits s. The indirect utility of an employed worker of type q is clearly decreasing in $\overline{q}$. As preferences are single peaked, the voting equilibrium will be the preferred allocation of the median $w^m = w^*(\frac{q^m}{\overline{q}}, \overline{q}, W)$ and $s^m = s^*(\frac{q^m}{\overline{q}})$. The indirect utility level of a employed worker at this political equlibrium outcome can be written as:$U^*(q, \overline{q}, W, q^m) = w^m + \delta q u(s^m)$.

Getting back to the first stage of the voting game, the preferred labor cost value W of a worker of type q will be the solution of the following program:

$$Max_W \quad \frac{L^D(W)}{M} U^*(q, \overline{q}, W, q^m) + (1 - \frac{L^D(W)}{M}) V^R(b, q)$$

$$R(L^D(W)) - W L^D(W) \geq \pi^R$$

Assuming that the participation constraint of the firm is always non binding, we easily get the solution for the optimal labor cost $\widetilde{W}(q, \overline{q}, q^m)$ at the tangency point E^m of the iso utility of a the union member and the labor demand curve $L^D(W)$ (figure 9). An increase in q induces flatter iso-utility curves of the trade union member as long as public benefits after unemployment T_b are less than occupational benefits s. Indeed in such a situation, the relative value of being employed for a worker increases with his type q. Hence his is more ready ot tradeoff a lower expected labor cost W for a higher probability of being employed. Consequently the cost of labor $\widetilde{W}(q, \overline{q}, q^m)$ and the employment level preferred by a union member of type q is decreasing in q (respectively increasing in q).From this and the single peakness of the expected utility function of a typical union member in W, it follows that the majority voting equilibrium in the union in the first stage will be again the one chosen by the median voter of the union q^m and the equilibrium discounted labor cost will be $W^m = \widetilde{W}(q^m, \overline{q}, q^m)$ out of which we deduce the employment level $L^m = L^D(W^m)$, and then the current wage $w^m = w^*(\frac{q^m}{\overline{q}}, \overline{q}, W^m)$ and benefits $s^m = s^*(\frac{q^m}{\overline{q}})$.

$>$From this analysis, one may investigate how changes, within the union, in the distribution of concerns for benefits $F(q)$ affects the equilibrium values of current wages, benefits and employment. Consider first that on average unions' members are more attached to the firm (ie. $\overline{q}$ increases). As the value of being employed decreases with $\overline{q}$, the slope of the iso-utility curves of the median member in figure 7 becomes steeper, leading to a higher expected cost of labor W^m picked up by the union and lower employment L^m. Holding q^m constant, the impact on benefits $s = s^m$ is unambiguously negative for occupational benefits are now, ceteris paribus, more costly to the firm. The effect on current wages w^m is a priori ambiguous but is likely to be positive if the income effect through W^m is large enough.

Consider now an increase in the pivotal agent q^m. This may be interpreted as the fact that the distribution $F(.)$ is more skewed towards concerns for benefits or that people more concerned or attached to the firm get more political power within the union. In that case, obviously the pivotal agent in the union wants higher long term benefits s^m. At the same time, as in the present framework, occupational benefits are attached to employment, the pivotal agent also cares more about employment. Consequently, his preferred level of labor costs W^m is reduced, current wages w^m are reduced and employment L^m is increased.

3.3 Insiders, last-in-first-out rules and soft landing plans

So far, an important aspect of the analysis is the fact that members of the union face all the same probability of being unemployed. In reality, this actually does not hold. Some workers within the union may typically enjoy insider positions within the firm and therefore may face very little risk to be fired under bad external conditions. Also, very often, firms and unions subscribe to seniority rules and last-in-first-out (LIFO) conventions. These mechanisms discriminate between young workers and more senior workers in terms of their risks to be unemployed and their access to occupational benefits. Another dimension of implicit employment discrimination across workers is the use of "soft landing" plans like early retirement plans, long term unemployment insurance and disability plans to mitigate unemployment problems. As these plans are mainly applied to senior and older workers, they induce a differential outside payoff according to age or attachment in the contingency where the firm does not employ the worker. This in turn affects differentially the preferred labor package that the worker would like to vote for in the union.

One can capture the above features in our framework in the following way. First, consider that for a given employment cost W, the probability to be employed for a worker of type q is simply a function $\Phi(q, W)$ such that:

$$\int \Phi(q, W) f(q) dq = L^D(W)$$

and satisfying the following conditions:

$$\text{a) } \frac{\partial \Phi(q,W)}{\partial W} \leq 0, \text{ b) } \frac{\partial \Phi(q,W)}{\partial q} \geq 0 \text{ c) } \frac{\partial^2 \Phi(q,W)}{\partial W \partial q} \geq 0$$

Condition a) simply states that the probability of being employed decreases with the expected labor cost of the firm. Condition b) captures the idea of seniority rules and last-in-first-out (LIFO) conventions in the sense that more attached workers (or senior workers) have a higher probability to keep their jobs than more mobile or young workers. Finally condition c) states that the sensitivity to labor cost of being employed decreases (in absolute value) with the degree of attachment of the worker to the firm. This captures in a certain way the idea that more attached or older workers are more likely to be insiders. Therefore their employment status is less sensitive to wages than that of junior (mobile) workers. Obviously, the specification includes the special case of a uniform probability of employment $L^D(W)/M$ when $\Phi(q,W)$ is independent from q.

The "soft landing" idea applied to senior workers can be captured by simply supposing that the intertemporal payoff of an unemployed worker V^R is designed to be a fraction of what an employed worker of type q would receive in equilibrium $V^*(q)$ (ie. $V^R = k(q)V^*(q)$ where the fraction $k(q)$ is increasing in q. The cost difference between V^R and what the unemployed worker would receive without "soft landing"' is "externalized" and financed outside the firm by general taxation on the rest of the economy.

Given this setting, one may again look at the voting equilibrium on wages, benefits and employment inside the union. Again one can solve the problem in two stages. The second stage provides as before for a given labor cost W, the optimal mix preferred by the pivotal agent q^m between current wages $w^m = w^*(\frac{q^m}{\overline{q}}, \overline{q}, W)$ and benefits $s^m = s^*(\frac{q^m}{\overline{q}})$. In the first stage, the determination of the preferred labor cost W for a worker of type q is given by the solution of the amended following maximization problem:

$$Max_W \quad \Phi(q,W)U^*(q,\overline{q},W,q^m) + (1-\Phi(q,W))V^R$$

$$R(L^D(W)) - WL^D(W) \geq \pi^R$$

$$\int \Phi(q,W)f(q)dq = L^D(W)$$

Assuming again that the participation constraint of the firm never binds, we easily get the preferred labor cost $\widetilde{W}(q,\overline{q},q^m)$ of a worker of type q.

Interestingly now this labor cost $\widetilde{W}(q, \overline{q}, q^m)$ need not be decreasing in q. On the contrary, because of seniority rules, more attached workers (with a higher q) have a higher probability to be employed and therefore value more an increase W than without such rules. Since their probability of employment is less sensitive to W, they are also less concerned about the impact of an increase of the labor cost W on their change of employment status. Finally, because of the possibility of "soft landing" plans, their reservation payoff level in the contingency of unemployment is increased, inducing them to demand more on the labor cost side. For all these reasons, it is very likely that workers more attached to the firm (larger q) will have higher preferred costs of labor $\widetilde{W}(q, \overline{q}, q^m)$. Under single peakness of the expected utility function of a union member in W, the majority voting equilibrium will be again the one chosen by the median voter of the union q^m and the equilibrium discounted labor cost will be $W^m = \widetilde{W}(q^m, \overline{q}, q^m)$ associated to an employment level $L^m = L^D(W^m)$, current wage $w^m = w^*(\frac{q^m}{\overline{q}}, \overline{q}, W^m)$ and benefits $s^m = s^*(\frac{q^m}{\overline{q}})$.

The main difference with the previous section, is in terms of the impact of a change of the distribution of characteristics of union workers. Consider for instance an increase in the pivotal agent q^m. As the pivotal agent in the union becomes more attached to the firm, he also gets more protection against the risk of unemployment through the seniority rules, FILO conventions and "soft landing" plans. Hence, his preferred labor cost $W^m = \widetilde{W}(q^m, \overline{q}, q^m)$ increases and the employment level $L^m = L^D(W^m)$ decreases. Current wages w^m and benefits are now both increased. The cost of such a strategy isd obviously paid by junior workers who have a disproportionately high probability of unemployment.

>From the previous discussion, it follows that the more likely seniority rules, LIFO conventions and "soft landing "plans are implemented in the economy, the more likely aging and shifts in the distribution of characteristics of unions towards senior workers implies higher labor costs, higher unemployment and larger occupational benefits. Clearly, going one step beyond, we might also expect the seniority rules, FILO conventions and "soft landing "plans to be themselves partly endogenous and influenced by the unions' activities. As these mechanisms are generally protecting the old or attached workers rather than the young workers, one may suspect that the same shift in the distribution towards seniority will make these rules more likely to be implemented, reinforcing therefore the conditions under which one will obtain high labor costs, high unemployment (especially among the young or less attached workers) and larger occupational benefits.

4 Endogenous membership

4.1 Endogenous membership and political economy considerations within unions

Another interesting issue concerns the membership evolution of the union. So far, this was fixed to a given size M. Suppose now that it is endogenous and that workers who are indexed by the characteristic q distributed uniformely on $[0, 1]$ decide to join by comparing their expected payoff inside the union to the reservation payoff V^0 they might get in some other non unionized sector of the economy with $V^0 > V^R(0)$. Clearly in order to be decide to join the union, each worker has to anticipate what will be the political equilibrium (W^m, L^m, w^m, s^m) inside the union. This depends obviously on the position of the pivotal agent within the union who in turn is determined by the type of individuals who decide to join the union (Booth (1984)). Hence an equilibrium with endogenous membership should determine jointly the political equilibrium decided within the union, the size of the union and the nature of the pivotal agent within the union.

More precisely, an individual of type q may expect to get a payoff

$$V^*(q) = \Phi(q, W^m)U^*(q, \overline{q}, W^m, q^m) + (1 - \Phi(q, W^m))V^R(q)$$

by joining the union. For given $\overline{q}$, and q^m, this payoff is increasing in q. Hence, ceteris paribus, if an individual of type q decides to join the union, all individuals of type $q' > q$ will also join the union. Define therefore by $q_{\min}$ the lowest level such that all workers with a characteristic $q > q_{\min}$ decide to join the union. Hence for given $\overline{q}, W^m, q^m$, $q_{\min}$ is determined by :

$$\Phi(q_{\min}, W^m)U^*(q_{\min}, \overline{q}, W^m, q^m) + (1 - \Phi(q_{\min}, W^m))V^R(q_{\min}) = V^0$$

When seniority rules, LIFO conventions and "soft landing" plans are applied within the union, it is quite likely that $q_{\min}$ is increasing in $\overline{q}$ and q^m. The more, on average, the union is constituted of workers with a high attachment or seniority characteristic q (ie. a high $\overline{q}$) and reflects the political interest of senior workers (ie. a high q^m), then the less it is interesting for a mobile or young worker with a low q to join that union and the higher the threshold level $q_{\min}$ above which workers decide to be members of the union (see the appendix). At the same time, given that q is uniformely distributed on $[0,1]$, we should have that:

$$q^m = \overline{q} = \frac{1 + q_{\min}}{2}$$

The resulting union equilibrium with endogenous membership is then described in figure 10 at the intersection of the curve QQ which describes the positive relationship between $q_{\min}$ and q^m and the line MM which describes the relationship $q^m = (1 + q_{\min})/2$. As is plotted on the picture the two curves may intersect more than once implying that there may be multiple equilbria in union membership, wages and benefits. The intuition for such a possibility is quite simple. When the union is mainly composed of workers with a low q, the political equilibrium within the union is more likely to reflect the preferences of these workers with high employment, relatively low wages and low benefits. Anticipating this, more workers of these types will join the union in the first place. At the same time however, one may have another equilibrium in which the union is mainly composed of senior workers with high values of q and protected from unemployment by seniority and LIFO rules. The political pivotal worker within the union is in this case more likely to reflect the preferences of these workers with high wages and benefits, at the cost of a high probability of unemployment for workers with a low q. This in turn obviously discourages union membership of the latter and supports a structure of union membership biased towards senior workers.

Appendix

Political equilibrium inside the union:

Consider first the second voting stage of this collective decision mechanism. For a given $L = L^D(W)$ and $W = w + \frac{1}{1+r} s\overline{q}$, the typical problem of a worker of type $q$can now be written as:

$$Max_{w,s} \quad w + \ \delta q u(s)$$

$$w + \frac{1}{1+r} s\overline{q} = W$$

The optimal wage and long term benefit are given by the standard marginal condition

$$u'(s) = \frac{1}{\delta(1+r)} \frac{\overline{q}}{q} \tag{3}$$

and the budget constraint. From this we find the current wage

$$w = w^*(\frac{q}{\overline{q}}, \overline{q}, W) = W - \frac{1}{1+r} \overline{q} u'^{-1}(\frac{1}{\delta(1+r)} \frac{\overline{q}}{q})$$

and benefits

$$s = s^* = u'^{-1}(\frac{1}{\delta(1+r)} \frac{\overline{q}}{q})$$

and the indirect utility function of an employed worker $U(w^*, s^*, q) = U^*(q, \overline{q}, W)$. An increase in q induces now a downward shift in the first order condition (3). This in turn leads to a lower current wage w^* and a higher longer term benefit s^*. Obviously the indirect utility of an employed worker of type q is increasing in q. An increase in the average value $\overline{q}$ has an ambiguous effect on the current wage and a negative impact on benefits s. The indirect utility of an employed worker of type q is then decreasing in $\overline{q}$.

As preferences are single peaked, the voting equilibrium will be the preferred allocation of the median, namely

$$\begin{aligned} w^m &= w^*(\frac{q^m}{\overline{q}}, \overline{q}, W) = W - \frac{1}{1+r} \overline{q} u'^{-1}(\frac{1}{\delta(1+r)} \frac{\overline{q}}{q^m}) \\ \text{and } s^m &= s^*(\frac{q^m}{\overline{q}}) = u'^{-1}(\frac{1}{\delta(1+r)} \frac{\overline{q}}{q^m}) \end{aligned}$$

The indirect utility level of a employed worker at this political equlibrium outcome can be written as:

$$\begin{aligned}
U^*(q,\overline{q},W,q^m) &= w^m + \ \delta q u(s^m) \\
&= W - \frac{1}{1+r}\overline{q}u'^{-1}(\frac{1}{\delta(1+r)}\frac{\overline{q}}{q^m}) + \ \delta q u[u'^{-1}(\frac{1}{\delta(1+r)}\frac{\overline{q}}{q^m})]
\end{aligned}$$

Getting back to the first stage of the voting game, the preferred labor cost value W of a worker of type q will be the solution of the following program:

$$Max_W \quad \frac{L^D(W)}{M}U^*(q,\overline{q},W,q^m) + (1-\frac{L^D(W)}{M})V^R(q)$$

$$R(L^D(W)) - WL^D(W) \geq \pi^R$$

Assuming that the participation constraint of the firm is always non binding, we easily get the solution for the optimal labor cost $\widetilde{W}(q,\overline{q},q^m)$ for a worker of type q as given by the following first order condition:

$$\frac{dL^D(W)}{dW}[U^*(q,\overline{q},W,q^m) - V^R(q)] + L^D(W) = 0 \qquad (4)$$

Simple differentiation of this condition gives that $\widetilde{W}(q,\overline{q},q^m)$ is decreasing in q as long as public benefits after unemployment T_b are less than occupational benefits s. It is increasing in $\overline{q}$ and increasing (resp. decreasing) in q^m when $q \geq q^m$ (resp. $q \leq q^m$). From this and the single peakness of the expected utility function of a typical union member in W, the majority voting equilibrium in the union in the first stage is again the one chosen by the median voter of the union q^m and the equilibrium discounted labor cost is $W^m = \widetilde{W}(q^m,\overline{q},q^m)$ from which we deduce the employment level $L^m = L^D(W^m)$, and then the current wage $w^m = W^m - \frac{1}{1+r}\overline{q}u'^{-1}(\frac{1}{\delta(1+r)}\frac{\overline{q}}{q^m})$ and benefits $s^m = u'^{-1}(\frac{1}{\delta(1+r)}\frac{\overline{q}}{q^m})$. ■

Endogenous membership:.

$q_{\min}$ is determined by :

$$V(q_{\min},\overline{q},W^m,q^m) = \Phi(q_{\min},W^m)U^*(q_{\min},\overline{q},W^m,q^m) + (1-\Phi(q_{\min},W^m))V^R(q_{\min}) = V^0$$

Differentiation provides:

$$V'_{q_{\min}} dq_{\min} = -V'_{\overline{q}}\ d\overline{q} - V_{q^m}\ dq^m - V'_{W^m}[\frac{\partial W^m}{\partial q^m}dq^m + \frac{\partial W^m}{\partial \overline{q}}d\overline{q}]$$

At the same time:

$$V'_{\overline{q}} = \Phi(q_{\min}, W^m)\frac{\partial U^*(q_{\min}, \overline{q}, W^m, q^m)}{\partial \overline{q}} < 0$$

$$V_{q^m} = \Phi(q_{\min}, W^m)\frac{\partial U^*(q_{\min}, \overline{q}, W^m, q^m)}{\partial q^m} < 0 \text{ for } q_{\min} < q^m$$

also, as $q_{\min} < q^m$ and that $\widetilde{W}(q_{\min}, \overline{q}, q^m) < \widetilde{W}(q^m, \overline{q}, q^m)$:

$$V'_{W^m} = \frac{\partial V(q_{\min}, \overline{q}, W^m, q^m)}{\partial W^m} < 0$$

from this and the fact that under seniority rules, $W^m = \widetilde{W}(q^m, \overline{q}, q^m)$ is increasing in $q^m$and $\overline{q}$:

$$\frac{\partial q_{\min}}{\partial \overline{q}} > 0 \text{ and } \frac{\partial q_{\min}}{\partial q^m} > 0$$

■